# Effects of the Dirac Sea on the Giant Monopole States

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**Abstract**

Effects of the Dirac sea on the excitation energy of the giant monopole states are investigated in an analytic way within the $\sigma-\omega$ model. The excitation energy is determined by the relativistic Landau-Migdal parameters, $F_0$ and $F_1$. Their analytic expressions are derived in the relativistic random phase approximation(RRPA) without the Dirac sea, with the Pauli blocking terms and with the full Dirac sea. It is shown that in the RRPA based on the mean field approximation the Pauli blocking terms should be included in the configuration space, according to the relativistic Landau theory. In the renormalized RRPA, the incompressibility coefficient becomes negative, if N$\bar{\text{N}}$ excitations are neglected.

*Key words:* Relativistic model; Effects of the Dirac sea; Giant resonance states; Landau-Migdal parameters
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The relativistic mean field approximation(RMFA) neglects the Dirac sea in the description of the nuclear ground state. Recently, however, it has been numerically shown that in the relativistic random phase approximation(RRPA) built on the RMFA, the monopole states cannot be well described without the Pauli blocking terms which express transitions between the Dirac sea and the occupied Fermi sea [1]. If the blocking terms are neglected, the excitation energies of the monopole states in the RRPA are much lower than those in the time-dependent relativistic mean field approximation [2].

The purpose of the present paper is to show in an analytic way the role of the Dirac sea in the excitation energy of the monopole states. We will

discuss the monopole states of nuclear matter in terms of the Landau-Migdal parameters using the $\sigma-\omega$ model. First, we will show that in the RRPA based on the RMFA, one should take into account the Pauli blocking terms in the configuration space. Second, the real effects of the Dirac sea will be discussed in the renormalized RRPA. It will be shown that N$\bar{\mathrm{N}}$ states yield essential effects on the excitation energy through the Landau-Migdal parameter, $F_0$.

The Landau-Migdal parameters, $F_0$ and $F_1$, are obtained by the second derivative of the total energy density with respect to the quasiparticle distribution. In the RMFA, they are given by [3,4]

$$F_0 = F_{\mathrm{v}} - \frac{1 - v_{\mathrm{F}}^2}{1 + a_{\mathrm{s}} F_{\mathrm{s}}} F_{\mathrm{s}}, \quad F_1 = -\frac{v_{\mathrm{F}}^2 F_{\mathrm{v}}}{1 + \frac{1}{3} v_{\mathrm{F}}^2 F_{\mathrm{v}}}, \tag{1}$$

where we have defined

$$F_{\mathrm{s}} = N_{\mathrm{F}} \left( \frac{g_{\mathrm{s}}}{m_{\mathrm{s}}} \right)^2, \quad F_{\mathrm{v}} = N_{\mathrm{F}} \left( \frac{g_{\mathrm{v}}}{m_{\mathrm{v}}} \right)^2, \tag{2}$$

$$N_{\mathrm{F}} = \frac{2 p_{\mathrm{F}} E_{\mathrm{F}}}{\pi^2}, \quad v_{\mathrm{F}} = \frac{p_{\mathrm{F}}}{E_{\mathrm{F}}}, \quad E_{\mathrm{F}} = (p_{\mathrm{F}}^2 + M^{*2})^{1/2}. \tag{3}$$

In the above equations, $g_{\mathrm{s}}$ and $g_{\mathrm{v}}$ stand for the Yukawa coupling constants, $m_{\mathrm{s}}$ and $m_{\mathrm{v}}$ the masses of the $\sigma$- and $\omega$-meson, respectively, and $p_{\mathrm{F}}$ and $M^*$ denote the Fermi momentum and the effective nucleon mass. $N_{\mathrm{F}}$ and $v_{\mathrm{F}}$ represent the relativistic density of states at the Fermi surface and the relativistic Fermi velocity. The factor, $a_{\mathrm{s}}$, in $F_0$ of Eq.(1) will play an essential role in later discussions, which is given by

$$a_{\mathrm{s}} F_{\mathrm{s}} = \frac{4}{(2\pi)^3} \left( \frac{g_{\mathrm{s}}}{m_{\mathrm{s}}} \right)^2 \int d^3p \, \frac{\boldsymbol{p}^2}{E_p^3} \theta_p, \quad E_p = (\boldsymbol{p}^2 + M^{*2})^{1/2}, \tag{4}$$

where $\theta_p$ denotes the step function, $\theta(p_{\mathrm{F}} - |\boldsymbol{p}|)$.

In the relativistic model, the excitation energy of the monopole states is expressed as [5],

$$E_{\mathrm{M}} = \left( \frac{K}{\epsilon_{\mathrm{F}} \langle r^2 \rangle} \right)^{1/2}, \tag{5}$$

where $\epsilon_{\mathrm{F}}$ denotes the Fermi energy and $\langle r^2 \rangle$ the root mean square radius of the nucleus. The incompressibility coefficient, $K$, is expressed in terms of the

above relativistic Landau-Migdal parameters,

$$K = \frac{3p_{\mathrm{F}}^2}{\epsilon_{\mathrm{F}}} \frac{1 + F_0}{1 + \frac{1}{3}F_1}. \tag{6}$$

Since $p_{\mathrm{F}}$ is determined by the nucleon density, and $\epsilon_{\mathrm{F}}$ is related to the nucleon binding energy, $E_{\mathrm{B}}$, and the free nucleon mass, $M$,

$$\epsilon_{\mathrm{F}} = E_{\mathrm{B}} + M, \tag{7}$$

the excitation energy of the monopole state is a function of $F_0$ and $F_1$.

In order to see the effects of the Pauli blocking terms on the monopole states, we derive the Landau-Migdal parameters according to the RRPA. We calculate the longitudinal RRPA correlation functions with and without the Pauli blocking terms. By comparing them with the correlation function of the Landau theory[6], we will obtain the expressions of the Landau-Migdal parameters in each approximation.

When following our previous papers [4,7], the mean field correlation function, $\Pi_{\mathrm{H}}$ [4,7], is given by the Fourier transform of the single-particle Green function, $G_{\mathrm{H}}$,

$$\Pi_{\mathrm{H}}(A, B; k) = -\frac{1}{2\pi i} \int d^4p \, \mathrm{Tr}[\Gamma_A G_{\mathrm{H}}(p + k)\Gamma_B G_{\mathrm{H}}(p)], \tag{8}$$

where $k$ denotes the four-momentum, $(k_0, \boldsymbol{k})$, and $A$ and $B$ the Fourier transform of the external field expressed with the mean field, $\psi_{\mathrm{H}}(\boldsymbol{x})$,

$$A(\boldsymbol{k}) = \int d^3x \exp(i\boldsymbol{k} \cdot \boldsymbol{x})\bar{\psi}_{\mathrm{H}}(\boldsymbol{x})\Gamma_A \psi_{\mathrm{H}}(\boldsymbol{x}), \tag{9}$$

$\Gamma_A$ being some 4×4 matrices. The sum of the ring diagrams in the RRPA for the $\sigma - \omega$ model is described as [4,7],

$$\delta\Pi_{\mathrm{RPA}}(A, B; k) = \frac{\chi_{\mathrm{s}}\tilde{\chi}_{\mathrm{v}}}{\det U_{\mathrm{L}}} \Pi_{\mathrm{H}}(A, \Lambda^a; k)(U_{\mathrm{L}})_{ab}\Pi_{\mathrm{H}}(\Lambda^b, B; k), \tag{10}$$

where the contraction should be carried out with respect to the superfix and suffix, $a, b = -1, 0$, and $\Lambda_{-1}$ and $\Lambda_0$ are given by Eq.(9) with $\Gamma_A = 1$ and $\gamma_0$, respectively. Moreover, $\chi_{\mathrm{s}}$ and $\tilde{\chi}_{\mathrm{v}}$ represent,

$$\chi_{\mathrm{s}} = \frac{1}{(2\pi)^3} \frac{g_{\mathrm{s}}^2}{m_{\mathrm{s}}^2 - k^2}, \quad \tilde{\chi}_{\mathrm{v}} = \frac{1}{(2\pi)^3} \frac{g_{\mathrm{v}}^2}{m_{\mathrm{v}}^2 - k^2} \frac{k^2}{\boldsymbol{k}^2}. \tag{11}$$

The explicit form of the 2×2 matrix, $U_{\rm L}$, in Eq.(10) depends on whether or not the Pauli blocking terms are included in the mean field correlation functions as discussed below.

The Green function, $G_{\rm H}$, is given by the sum of those for a single-particle, hole and antinucleon,

$$G_{\rm H} = G_{\rm p}(1-\theta_p) + G_{\rm h}\theta_p + G_{\bar{\rm N}}. \tag{12}$$

It is rewritten as a sum of the density-dependent and the Feynman part [7],

$$G_{\rm H} = G_{\rm D} + G_{\rm F}, \quad G_{\rm D} = \theta_p(G_{\rm h} - G_{\rm p}), \quad G_{\rm F} = G_{\rm p} + G_{\bar{\rm N}}. \tag{13}$$

Hence, $\Pi_{\rm H}$ is composed of the four terms like $G_{\rm D}G_{\rm D}$, $G_{\rm D}G_{\rm F}$, $G_{\rm F}G_{\rm D}$ and $G_{\rm F}G_{\rm F}$ [7]. In the RRPA based on the RMFA, the $G_{\rm F}G_{\rm F}$ term is neglected, which is divergent, while in the previous calculations [4], the $G_{\rm D}G_{\rm F}$ and $G_{\rm F}G_{\rm D}$ terms, which contain the Pauli blocking $\rm N\bar{N}$ excitations like $G_{\rm p}\theta_p G_{\bar{\rm N}}$, have been kept. Then we have obtained

$$U_{\rm L} = \begin{pmatrix} \chi_{\rm s}(1-\tilde{\chi}_{\rm v}\Pi_{\rm v}) & \chi_{\rm s}\tilde{\chi}_{\rm v}\Pi_{\rm sv} \\ \chi_{\rm s}\tilde{\chi}_{\rm v}\Pi_{\rm sv} & \tilde{\chi}_{\rm v}(1-\chi_{\rm s}\Pi_{\rm s}) \end{pmatrix}, \tag{14}$$

where the mean field correlation functions are defined as

$$\Pi_{\rm s} = \Pi_{\rm H}(\Lambda_{-1},\Lambda_{-1};k), \quad \Pi_{\rm v} = \Pi_{\rm H}(\Lambda_0,\Lambda_0;k), \tag{15}$$
$$\Pi_{\rm sv} = \Pi_{\rm H}(\Lambda_{-1},\Lambda_0;k) = \Pi_{\rm H}(\Lambda_0,\Lambda_{-1};k).$$

The Landau prescription of the correlation functions is obtained at the limit $k \to 0$. In this limit we have

$$\Pi_{\rm s} = (2\pi)^3 N_{\rm F}\{(1-v_{\rm F}^2)\Phi(x) - a_{\rm s}\}, \tag{16}$$

$$\Pi_{\rm v} = (2\pi)^3 N_{\rm F}\Phi(x), \quad \Pi_{\rm sv} = (2\pi)^3 N_{\rm F}(1-v_{\rm F}^2)^{1/2}\Phi(x), \tag{17}$$

where $\Phi(x)$ stands for the Lindhard function with $x = k_0/(|\boldsymbol{k}|v_{\rm F})$ [4]. Using these equations, we obtain the generalized dielectric function from the factor of Eq.(10) as

$$\frac{1}{\chi_{\rm s}\tilde{\chi}_v}{\rm det}U_{\rm L} = (1+a_{\rm s}F_{\rm s})\left\{1 + \left(F_{\rm v} - \frac{1-v_{\rm F}^2}{1+a_{\rm s}F_{\rm s}}F_{\rm s} - v_{\rm F}^2F_{\rm v}x^2\right)\Phi(x)\right\}. \tag{18}$$

In the Landau theory, Eq.(18) should be written as [6]

$$\frac{1}{\chi_{\mathrm{s}}\tilde{\chi}_{\mathrm{v}}}\mathrm{det}U_{\mathrm{L}} = c\left\{1 + \left(F_0 + \frac{F_1}{1+\frac{1}{3}F_1}x^2\right)\Phi(x)\right\}. \tag{19}$$

By comparing Eq.(18) with (19), we obtain the Landau-Migdal parameters which are the same as in Eq.(1).

Next we investigate the role of the Pauli blocking terms in the Landau-Migdal parameters. We calculate the mean field correlation functions neglecting the Pauli blocking terms and taking the only particle-hole states. The calculation of the mean field correlation functions is a little different from the one in taking the Pauli blocking terms, since the correlation functions are not Lorentz covariant, and the continuity equation is provided in a different way,

$$k_\mu \Pi_{\mathrm{H}}(A, \Lambda^\mu; k) = \langle[\Lambda_0(\boldsymbol{k}), A^\dagger(\boldsymbol{k})]\rangle, \tag{20}$$

where $\Lambda^\mu$ is given by replacing $\Gamma_A$ with $\gamma^\mu$ in Eq.(9), and the r.h.s. is related to the expectation value of the ground state as,

$$\langle\ |[\Lambda_0(\boldsymbol{k}), A^\dagger(\boldsymbol{k'})]|\ \rangle = \delta(\boldsymbol{k}-\boldsymbol{k'})\langle[\Lambda_0(\boldsymbol{k}), A^\dagger(\boldsymbol{k'})]\rangle. \tag{21}$$

In including the Pauli blocking terms, the r.h.s. of Eq.(20) is always vanished, but in neglecting them, it is not for $\Lambda_{1,2,3}$. Hence, the relationship between the correlation functions due to the time- and the longitudinal component of the $\omega$-meson is written in the frame, $\boldsymbol{k} = (|\boldsymbol{k}|, 0, 0)$, as

$$\Pi_{\mathrm{H}}(\Lambda_1, \Lambda_1; k) = \frac{k_0^2}{|\boldsymbol{k}|^2}\Pi_{\mathrm{H}}(\Lambda_0, \Lambda_0; k) - a_{\mathrm{v}}, \tag{22}$$

where the additional term, $a_{\mathrm{v}}$, comes from the r.h.s. of Eq.(20),

$$a_{\mathrm{v}} = \langle[\Lambda_0(\boldsymbol{k}), \Lambda^{1\dagger}(\boldsymbol{k})]\rangle/|\boldsymbol{k}|. \tag{23}$$

Because of this fact, the back flow effects due to the longitudinal $\omega$-meson exchange on $\delta\Pi_{\mathrm{RPA}}$ are not simply normalized as $\tilde{\chi}_{\mathrm{v}}$, and $U_{\mathrm{L}}$ in this case depends on $a_{\mathrm{v}}$,

$$U_{\mathrm{L}} = \begin{pmatrix} \chi_{\mathrm{s}}(1 + a_{\mathrm{v}}\chi_{\mathrm{v}} - \hat{\chi}_{\mathrm{v}}\Pi_{\mathrm{v}}) & \chi_{\mathrm{s}}\hat{\chi}_{\mathrm{v}}\Pi_{\mathrm{sv}} \\ \chi_{\mathrm{s}}\hat{\chi}_{\mathrm{v}}\Pi_{\mathrm{sv}} & \hat{\chi}_{\mathrm{v}}(1-\chi_{\mathrm{s}}\Pi_{\mathrm{s}}) \end{pmatrix}, \tag{24}$$

where we have defined

$$\chi_{\rm v}=\frac{1}{(2\pi)^3}\left(\frac{g_{\rm v}}{m_{\rm v}}\right)^2, \quad \hat{\chi}_{\rm v}=\tilde{\chi}_{\rm v}-a_{\rm v}\chi_{\rm v}^2. \tag{25}$$

At the limit, $k\rightarrow 0$, $\chi_{\rm v}a_{\rm v}$ becomes to be

$$\chi_{\rm v}a_{\rm v}(k\rightarrow 0)=-4\left(\frac{g_{\rm v}}{m_{\rm s}}\right)^2\int\frac{d^3p}{(2\pi)^3}\frac{2\boldsymbol{p}^2/3+M^{*2}}{E_p^3}\theta_p=-\frac{1}{3}v_{\rm F}^2F_{\rm v}. \tag{26}$$

Moreover, $\Pi_{\rm s}$ of the present case has not the term, $a_{\rm s}$, in Eq.(16), while $\Pi_{\rm v}$ and $\Pi_{\rm sv}$ are the same as in Eq.(17). As a result the generalized dielectric function in this case is given by

$$\begin{aligned}&\frac{1}{\chi_{\rm s}\hat{\chi}_{\rm v}}{\rm det}U_{\rm L}\\ &=\left(1-\frac{v_{\rm F}^2}{3}F_{\rm v}\right)\left\{1+\left(F_{\rm v}-F_{\rm s}(1-v_{\rm F}^2)-\frac{v_{\rm F}^2F_{\rm v}}{1-\frac{1}{3}v_{\rm F}^2F_{\rm v}}x^2\right)\Phi(x)\right\}.\end{aligned} \tag{27}$$

Finally comparison of the above equation with Eq.(19) provides us with the Landau-Migdal parameters in neglecting the Pauli blocking terms,

$$F_0=F_{\rm v}-(1-v_{\rm F}^2)F_{\rm s}, \quad F_1=-v_{\rm F}^2F_{\rm v}. \tag{28}$$

The difference between Eqs.(1) and (28) is very clear. $F_0$ and $F_1$ in Eq.(28) have no denominator. In order to obtain the correct expressions of $F_0$ and $F_1$ within the RMFA, thus we need to include the Pauli blocking terms in the configuration space of the RRPA.

In the Landau prescription, the denominators in Eq.(1) come from the self-consistent derivative of the effective mass and the baryon current with respect to the quasi-particle distribution, $n_i$ [3,5]. As to the effective mass in the RMFA,

$$M^*=M-\left(\frac{g_{\rm s}}{m_{\rm s}}\right)^2\frac{1}{V}\sum_i n_i\frac{M^*}{E_{p_i}}, \tag{29}$$

we have

$$\frac{\partial M^*}{\partial n_j}=-\frac{1}{V}\left(\frac{g_{\rm s}}{m_{\rm s}}\right)^2\frac{M^*}{E_{p_j}}-\left(\frac{g_{\rm s}}{m_{\rm s}}\right)^2\frac{1}{V}\sum_i n_i\frac{\boldsymbol{p}_i^2}{E_{p_i}^3}\frac{\partial M^*}{\partial n_j}, \tag{30}$$

$V$ being the nuclear volume. The coefficient of $\partial M^*/\partial n_j$ in the r.h.s. yields $a_{\rm s}F_{\rm s}$ in the denominator of $F_0$, as seen in Eq.(4). In the Green function formalism, the effective mass of the RMFA is written as

$$M^* = M + i\left(\frac{g_{\rm s}}{m_{\rm s}}\right)^2 \int \frac{d^4p}{(2\pi)^4}\,{\rm Tr}\,G_{\rm D}(p). \tag{31}$$

Hence the coefficient of $\partial M^*/\partial n_j$ in Eq.(30) is given by

$$a_{\rm s}F_{\rm s} = -i\left(\frac{g_{\rm s}}{m_{\rm s}}\right)^2 \int \frac{d^4p}{(2\pi)^4}\,{\rm Tr}\left(\frac{\partial G_{\rm D}(p)}{\partial M^*}\right). \tag{32}$$

On the other hand, we have shown that

$$a_{\rm s}F_{\rm s} = -\frac{1}{(2\pi)^3}\left(\frac{g_{\rm s}}{m_{\rm s}}\right)^2 \Pi_{\rm Pauli}(k=0), \tag{33}$$

where $\Pi_{\rm Pauli}$ represents the Pauli blocking terms in Eq.(8) for $\Gamma_A = \Gamma_B = 1$. Thus it is seen that in the Landau prescription of the RMFA, the derivative of $G_{\rm D}$ includes implicitly the Pauli blocking terms. Indeed, we can prove that

$$\frac{\partial G_{\rm D}(p)}{\partial M^*} = G_{\rm D}G_{\bar{\rm N}} + G_{\bar{\rm N}}G_{\rm D} + G_{\rm D}\frac{M^*}{p_0}\frac{\partial}{\partial p_0}. \tag{34}$$

If we integrate the r.h.s. over $p_0$, the Pauli blocking terms only remain.

The same discussion is possible for the denominator of $F_1$. The self-consistent derivative of the current, $\boldsymbol{j}$, as to the quasi-particle distribution provides [3],

$$\frac{\partial \boldsymbol{j}}{\partial n_j} = \frac{1}{V}\frac{\boldsymbol{p}_j}{E_{p_j}} - \frac{1}{3}v_{\rm F}^2 F_{\rm v}\frac{\partial \boldsymbol{j}}{\partial n_j}. \tag{35}$$

The coefficient of the second term yields the denominator of $F_1$. Using the Green function, the current of the RMFA is written as,

$$\boldsymbol{\Sigma} = \left(\frac{g_{\rm v}}{m_{\rm v}}\right)^2 \boldsymbol{j} = -\,i\left(\frac{g_{\rm v}}{m_{\rm v}}\right)^2 \int \frac{d^4p}{(2\pi)^4}\,{\rm Tr}(\boldsymbol{\gamma}G_{\rm D}(p')), \quad \boldsymbol{p}' = \boldsymbol{p} - \boldsymbol{\Sigma}. \tag{36}$$

Hence, the coefficient of the second term in Eq.(35) is given by

$$-\frac{1}{3}v_{\rm F}^2 F_{\rm v}\delta_{ij} = -\,i\left(\frac{g_{\rm v}}{m_{\rm v}}\right)^2 \int \frac{d^4p}{(2\pi)^4}\,{\rm Tr}\left(\gamma^j\frac{\partial G_{\rm D}(p')}{\partial \Sigma^i}\right)\Bigg|_{\boldsymbol{\Sigma}=0}. \tag{37}$$

We can show that the above derivative of $G_{\rm D}$ required in the RMFA is expressed by using the Pauli blocking terms as

$$\frac{\partial G_{\rm D}}{\partial \Sigma^i} = G_{\rm D}\gamma_i G_{\bar{\rm N}} + G_{\bar{\rm N}}\gamma_i G_{\rm D} - G_{\rm D}\frac{p'^i}{E_{p'}}\frac{\partial}{\partial p_0}. \tag{38}$$

Let us explore the effects of the Pauli blocking terms in more detail. The Pauli blocking terms reduce always the contribution of the $\sigma$-meson to $F_0$ through the factor $a_{\rm s}$ in Eq.(1), since $a_{\rm s}$ is positive, as seen in Eq.(4). On the other hand, the contribution of the $\omega$-meson to $F_0$ is not affected. Therefore, the value of $F_0$ becomes always smaller, and the incompressibility, $K$, is reduced according to Eq.(6), when the Pauli blocking terms are neglected. On the contrary, the absolute value of $F_1$ in Eq.(28), which has no denominator, becomes always larger, compared with the correct one in Eq.(1), so that $K$ is enhanced in neglecting the Pauli blocking terms.

We calculate the values of $F_0$, $F_1$ and $K$ using the following parameters [8] as an example,

$$\begin{aligned} M = 939, \quad m_{\rm s} = 520, \quad m_{\rm v} = 783 \ \ (\text{MeV}), \\ g_{\rm s}^2 = 109.626, \quad g_{\rm v}^2 = 190.431, \end{aligned} \tag{39}$$

which reproduce the nucleon binding energy, $E_{\rm B} = -15.75$ MeV at $p_{\rm F} = 1.30$ fm$^{-1}$. In this case, we have $M^* = 0.541M$, $v_{\rm F} = 0.451$ and $a_{\rm s} = 9.07 \times 10^{-3}$. These values provide us with

$$F_0 = 0.569, \quad F_1 = -1.151, \quad K = 544 \text{ MeV} \tag{40}$$

in taking the Pauli blocking terms, and

$$F_0 = -0.368, \quad F_1 = -1.866, \quad K = 357 \text{ MeV} \tag{41}$$

in neglecting the Pauli blocking terms. Thus both Landau-Migdal parameters are strongly affected by the Pauli blocking terms, and, in particular, $F_0$ changes its sign. Consequently, the value of $K$ is fairly reduced in neglecting the Pauli blocking terms. In $^{208}$Pb, the reduction amounts to about 2.7 MeV for the present parameters. This fact may be observed in ref. [1] by numerical calculations.

Now real effects of the Dirac sea should be explored with the fully renormalized RRPA, where the $G_{\rm F}G_{\rm F}$ term is also calculated in the Hartree correlation functions. Then, the Pauli exclusion principle in both the Fermi sea and the

Dirac sea is correctly taken into account. Such calculations based on the renormalized Hartree approximation(RHA) have been done by the present authors in ref. [9]. For complete discussions, we quote those results here. The Landau-Migdal parameters in this case are given by

$$F_0 = F_\omega - \alpha_{\rm ren} F_{\rm s}, \quad F_1 = -\frac{v_{\rm F}^2 F_\omega}{1 + \frac{1}{3} v_{\rm F}^2 F_\omega}, \tag{42}$$

where we have used the abbreviations:

$$F_\omega = N_{\rm F} \left( \frac{g_{\rm v}}{m_0} \right)^2, \quad \alpha_{\rm ren} = \frac{1 - v_{\rm F}^2}{1 + a_{\rm s} F_{\rm s} + a_{\rm D}}. \tag{43}$$

Formally the above equation is similar to Eq.(1), but the mass of the $\omega$-meson is replaced by the bare mass, $m_0$, in $F_\omega$ and the Dirac sea yields an additional effect, $a_{\rm D}$ in $\alpha_{\rm ren}$ [9].

As to $F_1$, essentially there is no additional effects from the Dirac sea. The renormalized correlation function from the $G_{\rm F} G_{\rm F}$ term due to the $\omega$-meson exchange disappears at the limit $k \to 0$ and has no contribution to $F_1$. Replacement of $m_{\rm v}$ by $m_0$ comes from the fact that the $\omega$-meson propagator is written in terms of the bare mass[9]. The value of $F_1$, however, depends on those of the Yukawa coupling constants used in the RHA. In order to reproduce the nucleon binding energy and the Fermi momentum mentioned before[9], the RHA requires $g_{\rm s}^2 = 66.117$ and $g_{\rm v}^2 = 79.927$. These values give $M^* = 0.7306M$ and $m_0 = 691.171$ MeV, so that we obtain $F_1 = -0.620$.

On the other hand, $F_0$ is strongly affected by the Dirac sea through $a_{\rm D}$ in Eq.(43). In using $v_{\rm F} = 0.3502$ of the present RHA, its value is much larger than that of $a_{\rm s} F_{\rm s}$,

$$a_{\rm D} = 0.405, \quad a_{\rm s} F_{\rm s} = 0.0296. \tag{44}$$

This Dirac sea effects reduce strongly the contribution of the $\sigma$-meson to $F_0$ through $\alpha_{\rm ren}$, and we have

$$F_0 = 0.676. \tag{45}$$

If $a_{\rm D}$ were neglected, then the value of $F_0$ would be $-1.56$, which means $K < 0$. This fact reflects that $\bar{\rm N}$-degrees of freedom play an important role to stabilize the nucleus in the RHA.

Finally we give two comments. First, since the restoring force of the giant

quadrupole states comes mainly from the distortion of the kinetic energy density, its excitation energy depends on $F_1$[5],

$$E_{\mathrm{Q}} = \left( \frac{6p_{\mathrm{F}}^2}{5\epsilon_{\mathrm{F}}^2 \langle r^2 \rangle} \frac{1}{1 + \frac{1}{3}F_1} \right)^{1/2} . \tag{46}$$

Thus, the Pauli blocking terms affect also the excitation energy of the quadrupole states in the RRPA based on th RMFA. Moreover, the Pauli blocking terms should be taken into account in the description of the center of mass motion, which requires the correct $F_1$ [5]. This fact has been observed in arguments on the spurious state of RRPA by Dawson and Furnstahl [10]. In the same way the Pauli blocking terms are necessary for discussions of the nuclear current or magnetic moments [11]. The isovector dipole states depend on $F_1$ and $F_1'$, which also require the Pauli blocking terms. The detail will be published in a separate paper.

Second, we note that Eqs.(5), (6) and (46) are formally the same as those in nonrelativistic models, except for $\epsilon_{\mathrm{F}}$ in their denominators [5]. In nonrelativistic models, it is replaced by the nucleon mass, $M$. However, they are related to each other as Eq.(7) through the nucleon binding energy which is negligible compared with the nucleon mass. Thus the relativistic correction to the excitation energies of the monopole and quadrupole states is less than 1% for $E_{\mathrm{B}} = -15.75$ MeV, if the values of the Landau-Migdal parameters in relativistic models are the same as in nonrelativistic ones.

In conclusion, in the relativistic random phase approximation(RRPA) based on the relativistic mean field approximation, the Pauli blocking terms should be taken into account for consistent descriptions of the Landau-Migdal parameters. The real effects of the Dirac sea on the excitation energy of the monopole states are studied in the RRPA built on the renormalized Hartree approximation. Then the nucleon-antinucleon states affect strongly the excitation energy of the monopole states through the Landau-Migdal parameter, $F_0$. The incompressibility coefficient becomes negative, if antinucleon-degrees of freedom are neglected. The Landau-Migdal parameter, $F_1$, is not affected formally by the renormalization.

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